1. A crime is committed by one of two suspects, A and B. Initially, there is equal evidence against both of them. In further investigation at the crime scene, it is found that the guilty party had a blood type found in 10% of the population. Suspect A does match this blood type, whereas the blood type of Suspect B is unknown. (a) Given this new information, what is the probability that A is the guilty party? (b) Given this new information, what is the probability that B’s blood type matches that found at the crime scene?

Answer:

we can use **Bayes' Theorem** for part (a) and consider the information for part (b). Let's tackle each part step by step:

**(a) Probability that A is the Guilty Party**

**Definitions and Given Information:**

* Let AAA be the event that Suspect A is the guilty party.
* Let BBB be the event that Suspect B is the guilty party.
* Let MMM be the event that the blood type found at the crime scene matches the blood type of the guilty party.

Given:

* Initially, there is equal evidence against both suspects, so:

P(A)=P(B)=0.5P(A) = P(B) = 0.5P(A)=P(B)=0.5

* The guilty party has a blood type found in 10% of the population:

P(M∣A)=0.10P(M | A) = 0.10P(M∣A)=0.10

* For Suspect B, since their blood type is unknown, we assume P(M∣B)=0.10P(M | B) = 0.10P(M∣B)=0.10 if B is guilty.

**Calculate P(M)P(M)P(M), the probability that the blood type found matches the guilty party’s blood type:**

Using the Law of Total Probability:

P(M)=P(M∣A)×P(A)+P(M∣B)×P(B)P(M) = P(M | A) \times P(A) + P(M | B) \times P(B)P(M)=P(M∣A)×P(A)+P(M∣B)×P(B) P(M)=0.10×0.5+0.10×0.5=0.05+0.05=0.10P(M) = 0.10 \times 0.5 + 0.10 \times 0.5 = 0.05 + 0.05 = 0.10P(M)=0.10×0.5+0.10×0.5=0.05+0.05=0.10

**Apply Bayes' Theorem to find P(A∣M)P(A | M)P(A∣M):**

P(A∣M)=P(M∣A)×P(A)P(M)P(A | M) = \frac{P(M | A) \times P(A)}{P(M)}P(A∣M)=P(M)P(M∣A)×P(A)​ P(A∣M)=0.10×0.50.10=0.050.10=0.50P(A | M) = \frac{0.10 \times 0.5}{0.10} = \frac{0.05}{0.10} = 0.50P(A∣M)=0.100.10×0.5​=0.100.05​=0.50

**Final Answer for (a):**

The probability that A is the guilty party given that the blood type found at the crime scene matches is **0.50** or **50%**.

**(b) Probability that B’s Blood Type Matches**

To find the probability that Suspect B’s blood type matches the one found at the crime scene, we consider the following:

**Definitions:**

* Let MBM\_BMB​ be the event that B’s blood type matches the blood type found at the crime scene.

We want to find P(MB∣B)P(M\_B | B)P(MB​∣B). Since Suspect B's blood type is unknown and we are assuming the guilty party has the matching blood type:

P(MB∣B)=P(M∣B)=0.10P(M\_B | B) = P(M | B) = 0.10P(MB​∣B)=P(M∣B)=0.10

**Final Answer for (b):**

The probability that Suspect B’s blood type matches the blood type found at the crime scene is **0.10** or **10%**.